

# Phase synchronization in tilted deterministic ratchets

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## Abstract

We study phase synchronization for a ratchet system. We consider the deterministic dynamics of a particle in a tilted ratchet potential with an external periodic forcing, in the overdamped case. The ratchet potential has to be tilted in order to obtain a rotator or self-sustained nonlinear oscillator in the absence of external periodic forcing. This oscillator has an intrinsic frequency that can be entrained with the frequency of the external driving. We introduced a linear phase through a set of discrete time events and the associated average frequency, and show that this frequency can be synchronized with the frequency of the external driving. In this way, we can properly characterize the phenomenon of synchronization through Arnold tongues, which represent regions of synchronization in parameter space, and discuss their implications for transport in ratchets.

*Key words:* Synchronization; Ratchets; Brownian Motors; Classical Transport

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## 1 Introduction

The phenomenon of synchronization is widespread in Nature. We witness its manifestations in many different places and contexts. Synchronization is essentially a nonlinear phenomenon and is very common in many complex systems, not only in the physical sciences, but in the life sciences as well [1,2,3,4]. In

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particular, the case of phase synchronization establishes a common formalism to treat both nonlinear periodic oscillators, as well as chaotic and noisy oscillators [1,5].

In a different context, there has been an increasing interest during recent years in the study of transport phenomena of nonlinear systems that can extract usable work from unbiased non-equilibrium fluctuations. These, so called Brownian motors (or thermal ratchets) can be modeled by a Brownian particle undergoing a random walk in a periodic asymmetric potential, and being acted upon by an external time-dependent force of zero average. The recent burst of work is motivated by both, (i) the challenge to model unidirectional transport of molecular motors within the biological realm and, (ii) the potential for novel technological applications that enables an efficient scheme to shuttle, separate and pump particles on the micro- and even nanometer scale [6,7,8,9,10,11].

Although the vast majority of the literature in this field considers the presence of noise, there have been attempts to model the transport properties of classical deterministic ratchets as well [12,13,14,15,16,17,18,19]. In this paper we will be dealing with a deterministic tilted ratchet in the overdamped regime that acts as a rotator or self-sustained oscillator with a characteristic frequency, even in the absence of an external periodic forcing. The dynamics can be represented by a particle in a washboard potential that has been studied in many different contexts, like phase dynamics in synchronization [1,5], pendulum dynamics [20], rotators [1], superionic conductors [21], optical potentials [22], excitable systems [23], diffusion on surfaces [24,25,26,27,28,29], charge density waves [30] and Josephson junctions dynamics [1,21,31]. When the washboard potential is periodically driven it exhibits a great variety of nonlinear phenomena including phase locking, hysteresis [32] and chaos [33].

Here we will study the synchronization properties of an overdamped particle moving on a tilted ratchet potential that is rocked by a periodic external force. Throughout this paper we will consider a constant force above the critical value, in such a way that the particle slides down the washboard potential, even though it is in the overdamped regime. When the periodic forcing is absent, the particle experiences only a fixed washboard potential, and moves through each period of the ratchet in a given constant time that defines the period  $\tau_0$  of this rotator. The associated frequency  $\omega_0 = 2\pi/\tau_0$  is its characteristic frequency. In this sense, this rotator is effectively acting as a self-sustained oscillator, with its own characteristic frequency. That is, if the rotator is driven by a constant force, it acquires the same features of a self-sustained oscillator, having a limit cycle in phase space. Thus, forced rotators are similar to self-sustained oscillators and can be synchronized by a periodic external force. We will drive this rotator with an external periodic force of period  $\omega_D$ . In this way we can define properly the synchronization of the rotator and the external forcing.

The current or average velocity, which is the important quantifier for this system, displays steps as a function of a control parameter. This last result has been found previously by other authors that have studied overdamped deterministic ratchets [12,13,14,34]. On the experimental side, these so called Shapiro steps have been found recently for deterministic Josephson vortex ratchets and three-junction SQUID rocking ratchets [35,36].

More recent studies consider the problem of synchronization of deterministic ratchets, but they deal with complete synchronization between two coupled ratchets [37,38,39,40], and with anticipated synchronization between two unidirectional coupled ratchets with time delay [41]. In this work, instead, we are dealing with phase synchronization through a linear phase, properly defined through a set of discrete time events.

As a way of characterizing the synchronization phenomenon, we will calculate, for the first time, the so called Arnold tongues for the tilted deterministic ratchet. For a description of Arnold tongues in circle maps and pendulum dynamics see [20,30,42]. Arnold tongues are regions of synchronization in a parameter space. Here we will calculate these regions in a two-dimensional parameter space defined by the ratio  $\omega_D/\omega_0$  and the amplitude of the driving periodic force  $F_D$ . The tips of the tongues are located on rational values of the ratio  $\omega_D/\omega_0 = p/q$ , where  $p$  and  $q$  are integer numbers. Each tongue is therefore labeled by a rational  $p/q$  whose inverse is precisely the value of the current in the driven washboard potential; the widths of these Arnold tongues correspond to the size of the steps of the current as a function of  $F_D$ .

## 2 Tilted ratchets as nonlinear rotators

To start out, let us consider now the one-dimensional problem of a particle driven by a periodic time-dependent external force in an asymmetric periodic ratchet potential. Here, we do not take into account any sort of noise, meaning that the dynamics is deterministic. Two additional forces act on the particle: a dissipative force proportional to velocity, and an external constant force. We thus deal with a rocked deterministic tilted ratchet[12,14] in the overdamped limit that obeys the following equation of motion:

$$m\gamma\dot{x} + \frac{dV(x)}{dx} = F + F_D \cos(\omega_D t), \quad (1)$$

where  $m$  is the mass of the particle,  $\gamma$  is the friction coefficient,  $V(x)$  is the asymmetric periodic ratchet potential,  $F$  is a constant force,  $F_D$  and  $\omega_D$  represent the amplitude and the frequency of the external driving force, respectively.

The ratchet potential is given by

$$V(x) = V_0 \left[ C - \sin \frac{2\pi(x - x_0)}{L} - \frac{1}{4} \sin \frac{4\pi(x - x_0)}{L} \right], \quad (2)$$

where  $L$  is the periodicity of the potential,  $V_0$  is the amplitude, and  $C$  is an arbitrary constant. The potential is shifted by an amount  $x_0$  in order that the minimum of the potential is located at the origin [16].

Let us define the following dimensionless units:  $x' = x/L$ ,  $x'_0 = x_0/L$ ,  $t' = \gamma t$ ,  $\omega'_D = \omega_D/\gamma$ ,  $F' = F/mL\gamma^2$ ,  $F'_D = F_D/mL\gamma^2$ ,  $V' = V/mL^2\gamma^2$  and  $V'_0 = V_0/mL^2\gamma^2$ . Thus, we are using the periodicity of the potential  $L$  as the natural length scale and the inverse of the friction coefficient  $\gamma$  defines the natural time scale. With these two quantities, the natural force is given by  $mL\gamma^2$  and the associated energy by  $mL^2\gamma^2$ .

The dimensionless equation of motion, after renaming the variables again without the primes, becomes

$$\dot{x} + \frac{dV(x)}{dx} = F + F_D \cos(\omega_D t), \quad (3)$$

where the dimensionless potential can be written as

$$V(x) = V_0 \left[ C - \sin 2\pi(x - x_0) - \frac{1}{4} \sin 4\pi(x - x_0) \right] \quad (4)$$

and is depicted in the inset of Fig. 1. The constant  $C$  is such that  $V(0) = 0$ , and is given by  $C = -(\sin 2\pi x_0 + 0.25 \sin 4\pi x_0)$ . We choose,  $x_0 \simeq -0.19$

We can rewrite the equation of motion Eq. (3) as

$$\dot{x} + \frac{\partial U(x, t)}{\partial x} = 0, \quad (5)$$

where  $U(x, t) = V(x) - [F + F_D \cos(\omega_D t)]x$ . The important point to stress here is that we need to add a constant force  $F$  to the ratchet in order to tilt the ratchet potential and, in this way, obtain a rotator (that acts as a self-sustained oscillator), even without the external periodic forcing [1,44]. In this way, we can properly synchronize the characteristic frequency of the rotator with the driving frequency  $\omega_D$ .

When  $F_D = 0$ , we have a tilted ratchet that obeys the equation of motion:  $\dot{x} + dV(x)/dx = F$ . The tilted (time-independent) washboard potential is, in this case,  $U(x) = V(x) - Fx$ , see Fig. 1. Thus, this ratchet becomes a rotator

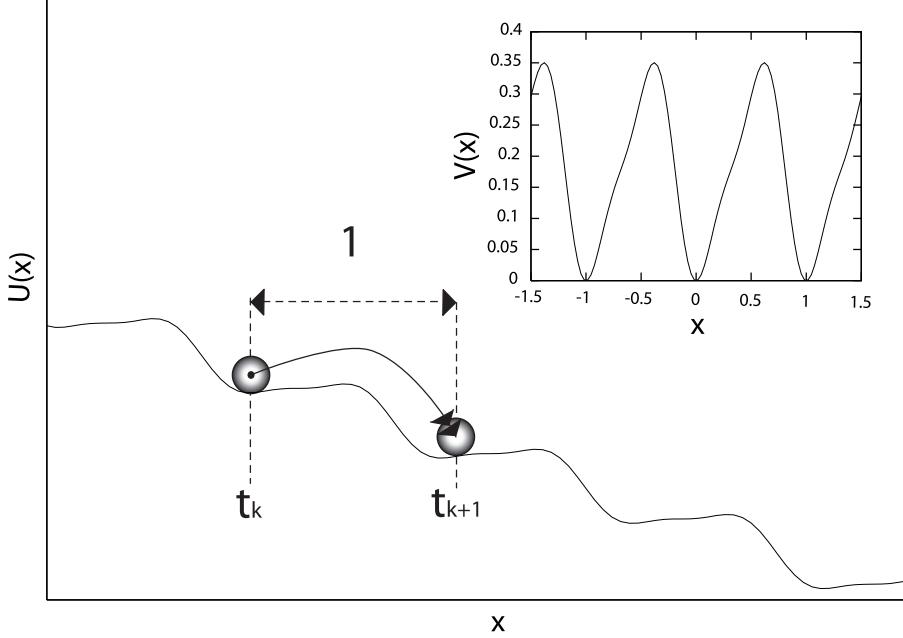


Fig. 1. The tilted washboard potential indicating the dynamics that defines the discrete time events  $t_k$ . The ratchet potential without tilt is illustrated in the inset. that has a characteristic frequency  $\omega_0$ . The associated period of the rotator  $\tau_0 = 2\pi/\omega_0$  can be obtained directly by integrating this equation of motion. However, there is another way to obtain  $\omega_0$  for this tilted ratchet that relays on the introduction of a phase variable for this rotator. So, in what follows we will introduced this general concept that we will use in the rest of the paper.

In order to define a phase variable we need first to obtain a discrete process from the continuous dynamics by introducing discrete time events. These discrete times can be defined as the times when the particle arrives at the discrete position  $x_k = \pm k$ , which correspond to the minima of the ratchet potential without tilt. Here  $k = 0, 1, 2, \dots$ . Remember that the period of the ratchet is one:  $V(x+1) = V(x)$ . This defines the set of times  $t_k$ , where  $k$  is a nonnegative integer. In Fig. 1 we show the washboard potential illustrating these discrete times. Once we obtain these set of markers, we can define an instantaneous linear phase for the rotator as [1,5]

$$\phi(t) = 2\pi \frac{t - t_k}{t_{k+1} - t_k} + 2\pi k, \quad t_k \leq t < t_{k+1} \quad (6)$$

This linear phase is valid in the indicated interval and defines a piecewise linear function of time that increases by  $2\pi$  each time the particle crosses the dimensionless position  $x_k = \pm k$ .

Given this phase, we can define the instantaneous frequency of the rotator as  $\omega(t) = \dot{\phi}(t)$ , and the average frequency as

$$\langle \omega \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \omega(t) dt = \lim_{T \rightarrow \infty} \frac{1}{T} [\phi(T) - \phi(0)]. \quad (7)$$

Without loss of generality, we choose  $t_0 = 0$ , and thus  $\phi(0) = 0$ . The limit above can be written as

$$\langle \omega \rangle = \lim_{k \rightarrow \infty} \frac{\phi(t_k)}{t_k} = 2\pi \lim_{k \rightarrow \infty} \frac{k}{t_k} \quad (8)$$

This is the simplest way to calculate the average frequency; we simply count the number of jumps (given by  $k$ ) and divide by the total time span  $t_k$ .

The other quantity of importance is the average velocity or current in the tilted ratchet. In order to evaluate this current we have to calculate the number  $k$  of unit periods that the particle crosses to the right, denoted by  $N_k^R$ , and the number of crossings to the left, given by  $N_k^L$ . The total number of periods traversed on the ratchet is given by  $N_k^T = N_k^R + N_k^L$ . The difference

$$N_k = N_k^R - N_k^L \quad (9)$$

indicates that during the time  $t_k$  the particle has covered the distance  $x_k = N_k$ .

Therefore, the average velocity (current) is given by

$$\langle v \rangle = \lim_{k \rightarrow \infty} \frac{N_k}{t_k} \quad (10)$$

In the particular case when  $N_k^L = 0$ , that is, when there are no jumps to the left, we have  $N_k = N_k^R = k$ . Thus,

$$\langle v \rangle = \lim_{k \rightarrow \infty} \frac{k}{t_k} = \frac{1}{2\pi} \langle \omega \rangle \quad (11)$$

In the simple case of a tilted ratchet without external forcing,  $F_D = 0$ , the average frequency defined above coincides with the natural frequency of the rotator, that is,  $\langle \omega \rangle = \omega_0$ .

The above treatment is quite general and can be used in the case of a tilted ratchet with an inertial term, even though this inertial ratchet can display a chaotic dynamics [15,16,17,18,19], and also in the case of a tilted ratchet

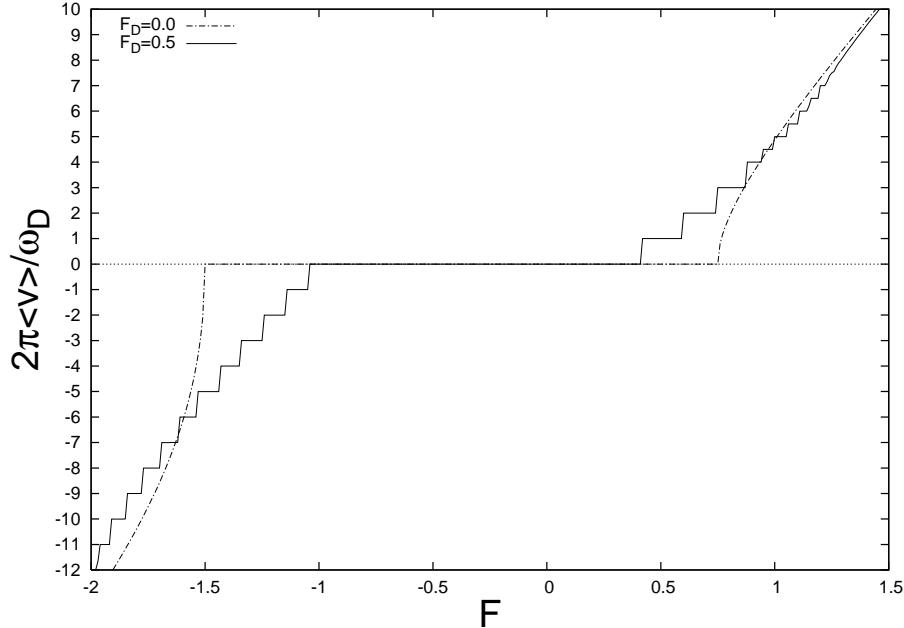


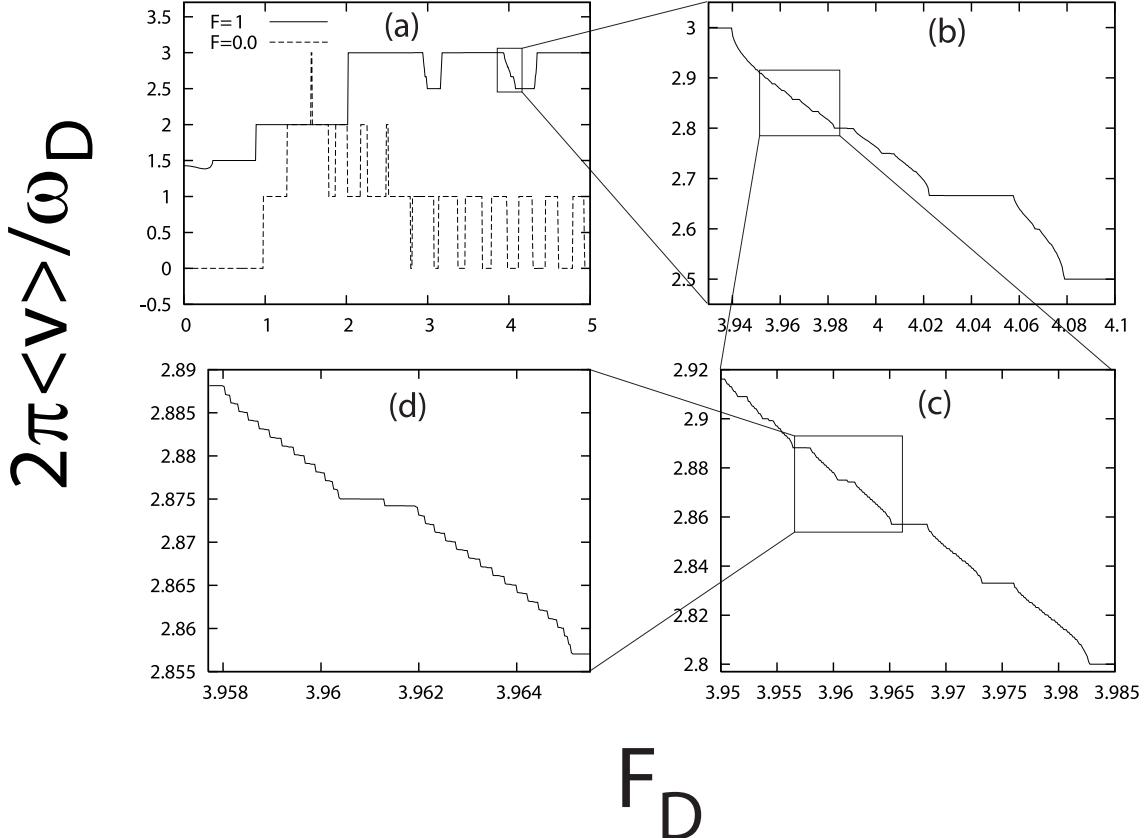
Fig. 2. The average velocity as a function of the external force  $F$ . The dashed line indicates the case when the periodic driving is absent ( $F_D = 0$ ) and the continuous line is the case when the periodic driving is acting on the particle with an amplitude  $F_D = 0.5$ . In both cases we used  $\omega_D = 0.7$ .

with noise. We have used the concept of a linear phase due to its broad-range applications to the cases of periodic and chaotic oscillators [1], chaotic rotators [44], and oscillators in the presence of noise [5]. Additionally, the introduction of the discrete events that define the linear phase allow us to simplify the dynamics and have a more clear picture of the synchronization involved.

### 3 Numerical results

In this section we will solve numerically the equation of motion for the rocking tilted ratchet. We use the fourth-order Runge-Kutta algorithm to solve the differential equation Eq. (3). Once we obtain the full trajectory, we identify the set of discrete times  $t_k$  when the particle crosses the positions  $x_k$ . With this marker events we calculate directly the average frequency using Eq. (8) and, after calculating the quantity  $N_k$ , we obtain the current, using Eq. (10). We will fix throughout the paper the amplitude of the ratchet potential as  $V_0 = 1/2\pi$ . With this value, the critical tilt to the right is  $F_c^R = 0.75$  and to the left  $F_c^L = -1.5$ .

In Fig. 2, we depict the average velocity, scaled with the driving frequency,  $2\pi\langle v \rangle / \omega_D$  as a function of the tilt  $F$ . The dashed line shows the case without periodic driving ( $F_D = 0$ ) and corresponds to the fixed washboard potential



$F_D$

Fig. 3. The scaled average velocity  $2\pi\langle v \rangle/\omega_D$  as a function of the amplitude of the periodic driving  $F_D$ . (a) The dashed line indicates the case when the tilt is absent ( $F = 0$ ) and the continuous line is the case when the tilt is  $F = 1$ . (b), (c) and (d) correspond to successive magnifications of the current for the tilted ratchet, showing a self-similar structure of steps, typical of a devil's staircase. In the dashed line in (a), we used  $\omega_D = 0.7$ , since in this case  $\omega_0$  is not defined. In the other cases we used  $\omega_D = 0.7\omega_0$ .

$U(x) = V(x) - Fx$ . Notice that the current is zero until we arrive at the critical tilt  $F_c^R = 0.75$  to the right or to the critical tilt  $F_c^L = -1.5$  to the left. This step of zero current is not centered around the origin, due to the asymmetry of the ratchet potential. For values greater than  $F_c^R$  we have a finite current that increases monotonically with  $F$ . Of course, for values less than  $F_c^L$  we obtain a negative average velocity that decreases for negative values of the tilt. When the periodic driving is present, this scaled current acquires a series of clearly defined steps for values of the current given by the ratio  $p/q$ , where  $p$  and  $q$  are integer numbers. In many cases,  $q = 1$  and the average scaled current is an integer. In the context of Josephson junctions, these are the celebrated Shapiro steps [33].

Remember that in the case where all the jumps are to the right, that is,  $N_k^L = 0$ , we show that  $2\pi\langle v \rangle = \langle \omega \rangle$ . Therefore, a rational value of  $2\pi\langle v \rangle/\omega_D = p/q$  means that  $\langle \omega \rangle = (p/q)\omega_D$  for a whole range of values of the tilt. This

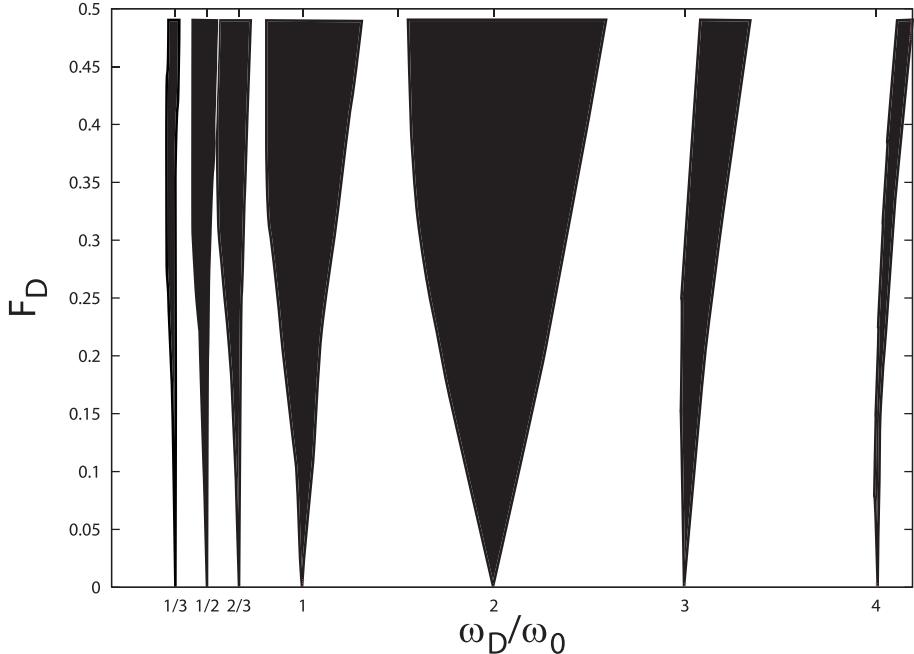


Fig. 4. Arnold tongues in the parameter space  $F_D$  against  $\omega_D/\omega_0$ . Notice that the tongues are located precisely at the rational values  $p/q$ , for  $p$  and  $q$  integer numbers, of the ratio between the driving frequency and the natural frequency of the rotor. Here the tilt is  $F = 1$  and the corresponding frequency is  $\omega_0 \simeq 3.41$ . Each tongue is labeled by the inverse  $q/p$  that gives the value of the scaled average velocity  $2\pi\langle v \rangle/\omega_D$  in that region of the parameter space.

phenomenon is called frequency locking.

In Fig. 3, we show the scaled average velocity  $2\pi\langle v \rangle/\omega_D$  as a function of the amplitude of the periodic driving  $F_D$ . In (a), the dashed line depicts the current for the ratchet without tilt ( $F = 0$ ) and coincide with previous calculations [12,13,14] showing a structure of steps of unit height. The solid line shows the current for a tilted ratchet with  $F = 1$  that also has well defined steps for rational values. In (b), (c) and (d) we show successive magnifications of the current that clearly exhibits a self-similar structure of steps, typical of a devil's staircase [42,43]. This detailed structure has been reported before [13,14] for an overdamped ratchet without tilt, but here we obtained this fractal current with a devil's staircase also for the tilted ratchet.

In Fig. 4, we depict the parameter space  $F_D$  against  $\omega_D/\omega_0$  that shows regions of synchronization, called Arnold tongues, located at rational values  $p/q$ , where  $p$  and  $q$  are integer numbers. We choose a tilt  $F = 1$ , which corresponds to  $\omega_0 \simeq 3.41$ . Notice that the tongues start, for small values of  $F_D$ , precisely at these rational values, as indicated in the figure. Each Arnold tongue corresponds to one particular rational  $p/q$  whose inverse gives the value of the scaled average velocity  $2\pi\langle v \rangle/\omega_D$  in that region of the parameter space. This is depicted in a three-dimensional plot in Fig. 5. Therefore, the average velocity, properly

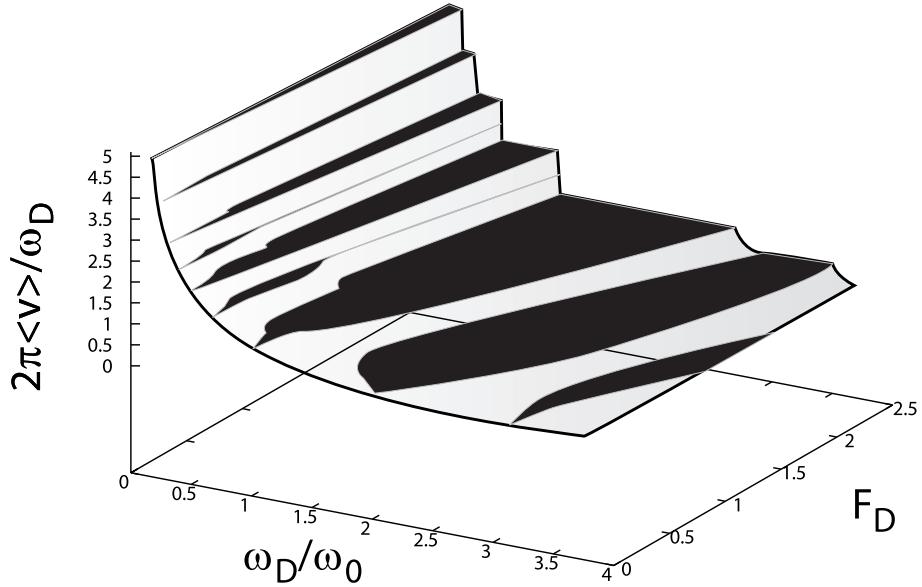


Fig. 5. Three-dimensional plot of the scaled average velocity  $2\pi\langle v \rangle/\omega_D$  as a function of  $F_D$  and  $\omega_D/\omega_0$ . A projection of this 3D plot shows the Arnold tongues in the parameter space of Fig. 4. Here the tilt is  $F = 1$  and  $\omega_0 \simeq 3.41$ .

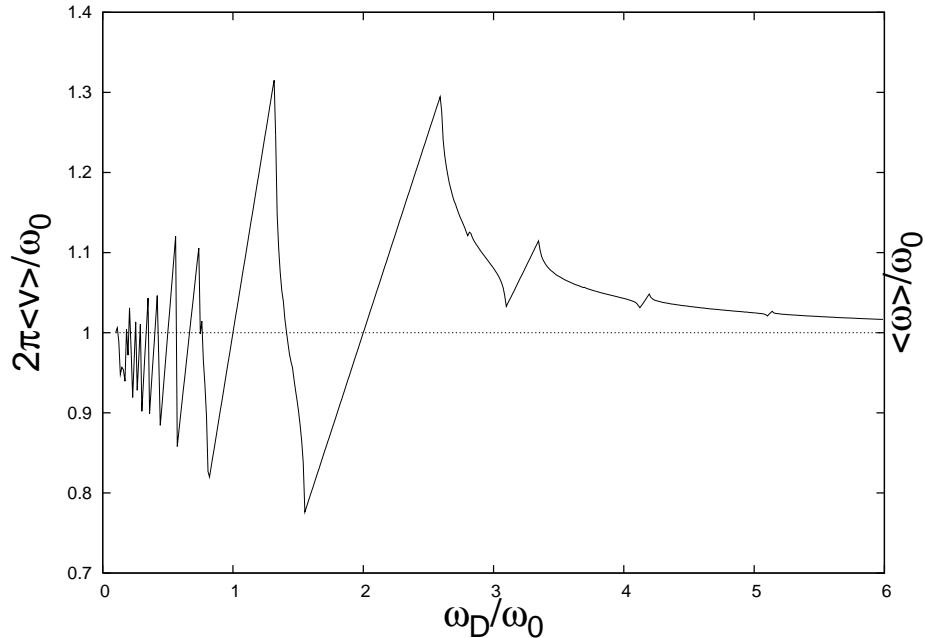


Fig. 6. Average velocity, scaled now as  $2\pi\langle v \rangle/\omega_0$ , and the average frequency of the rotator, scaled as  $\langle \omega \rangle/\omega_0$  as a function of the ratio  $\omega_D/\omega_0$ . The straight line segments correspond to the steps observed due to phase synchronization in Fig. 4. The peaks in the current are located at the right borders of the Arnold tongues in Fig. 4. Here the tilt is  $F = 1$ ,  $\omega_0 \simeq 3.41$ , and  $F_D = 0.5$ .

scaled, acquires rational values  $q/p$ , which correspond to the phenomenon of phase synchronization in this forced tilted ratchet, acting as a rotator [44].

Finally, in Fig. 6, we plot the average velocity, scaled now as  $2\pi\langle v \rangle / \omega_0$ , as a function of the ratio  $\omega_D / \omega_0$ . Here  $\omega_0$  is a fixed value that correspond to the characteristic frequency of the tilted ratchet in the absence of periodic driving. In the same figure, we plot the average frequency of the rotator, scaled as  $\langle \omega \rangle / \omega_0$ , calculated using the discrete dynamics explained in the previous section. Instead of plateaus, we have now straight lines, since we are scaling the current with a fixed parameter  $\omega_0$ , instead of the running parameter  $\omega_D$ . In this case, all the jumps are to the right direction and therefore  $2\pi\langle v \rangle = \langle \omega \rangle$ , that is, the average velocity is proportional to the average frequency of the rotator.

Notice that the peak values of the average velocity correspond to the borders of the Arnold tongues in Fig. 4. Thus, when we are crossing an Arnold tongue (synchronization region) the current increases linearly with  $\omega_D / \omega_0$ ; at the right border of the tongue the current is maximal and outside the tongue starts to decrease until we arrive at the next tongue to increase linearly again, and so forth. At the right border of the Arnold tongue, labeled by  $p/q = 1$ , the current has a maximum, followed by the second largest peak at the right border of the tongue with  $p/q = 2$ . The heights of the peaks in the current arise due to the combined effect of both the width of the tongues and the slope of the linear segments. The slopes of each of the segments correspond precisely to the inverse values  $q/p$  that label the tongues. Therefore, the peaks in the current are associated with the phenomenon of synchronization.

#### 4 Concluding remarks

In summary, we have analyzed the phenomenon of phase synchronization in tilted deterministic ratchets in the overdamped regime and with an external periodic forcing. The dynamics in this rocked washboard potential corresponds precisely with the dynamics of a rotator that, acting as a self-sustained oscillator, can be capable of being synchronized with the external periodic drive. We can clearly identify three frequencies for this system: the characteristic frequency of the rotator without driving, the driving frequency itself, and the average frequency of the rotator with driving. This average frequency is the derivative of a time-dependent phase, that can be obtained through a set of discrete time events and is a piecewise linear function of time between these markers. We calculated the average frequency and the average velocity as a function of the tilt and obtained the well-known Shapiro steps that characterize the phenomenon of frequency locking. We also exhibit the self-similar structure of steps in the current, typical of a devil's staircase. We obtained well-defined Arnold tongues in the 2D parameter space given by the amplitude and the frequency of the periodic forcing. Each Arnold tongue is labeled by a rational number  $p/q$ , where  $p$  and  $q$  are integer numbers, whose inverse gives precisely

the rational value of the average scaled velocity of the particle. Finally, we show that the local maxima in the average velocity correspond to the borders of these Arnold tongues and, in this way, we established a connection between optimal transport in ratchets and the phenomenon of phase synchronization.

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## References

- [1] A. Pikovsky, M. Rosenblum, J. Kurths, *Synchronization. A universal concept in nonlinear sciences*, Cambridge University Press, Cambridge, 2001.
- [2] L. Glass, *Nature* 410 (2001) 277.
- [3] S. Boccaletti, J. Kurths, G. Osipov, D. L. Valladares, C. S. Zhou, *Phys. Rep.* 366 (2002) 1.
- [4] J. A. Acebrón, L. L. Bonilla, C. J. Pérez Vicente, F. Ritort, R. Spigler, *Rev. Mod. Phys.* 77 (2005) 137.
- [5] V. S. Anishchenko, V. V. Astakhov, A. B. Neiman, T. E. Vadivasova, L. Schimansky-Geier, *Nonlinear Dynamics of Chaotic and Stochastic Systems*, Springer, Berlin, 2002.
- [6] R. D. Astumian, P. Hänggi, *Physics Today* 55, No. 11 (2002) 33.
- [7] P. Reimann, *Phys. Rep.* 361 (2002) 57.
- [8] P. Reimann, P. Hänggi, *Applied Physics A* 75 (2002) 169.
- [9] H. Linke, *Appl. Phys. A* 75 (2002) 167. Special issue on Brownian motors.
- [10] P. Hänggi, F. Marchesoni, F. Nori, *Ann. Physik (Leipzig)* 14 (2005) 51.
- [11] J. Klafter, M. Urbakh, *J. Phys: Condens. Matter* 17, No. 47 (2005). Special issue on molecular motors.
- [12] R. Bartussek, P. Hänggi, J. G. Kissner, *Europhys. Lett.* 28 (1994) 459.
- [13] A. Ajdari, D. Mukamel, L. Peliti, J. Prost, *J. Phys. I France* 4 (1994) 1551.
- [14] P. Hänggi, R. Bartussek, in “*Nonlinear Physics of Complex Systems*”, J. Parisi, S. C. Müller, W. Zimmermann, eds., *Lecture Notes in Physics* Vol. 476, pp. 294-308, Springer, Berlin, 1996.
- [15] P. Jung, J. G. Kissner, P. Hänggi, *Phys. Rev. Lett.* 76 (1996) 3436.

- [16] J. L. Mateos, Phys. Rev. Lett. 84 (2000) 258.
- [17] J. L. Mateos, Physica D 168-169 (2002) 205.
- [18] J. L. Mateos, Physica A 325 (2003) 92, and references therein.
- [19] M. Borromeo, G. Costantini, F. Marchesoni, Phys. Rev. E 65 (2002) 041110.
- [20] G. L. Baker, J. P. Gollub, *Chaotic Dynamics*, Cambridge University Press, Cambridge, 1990.
- [21] H. Risken, *The Fokker-Planck Equation*, Springer, Berlin, 1996.
- [22] S. A. Tatarkova, W. Sibbett, K. Dholakia, Phys. Rev. Lett. 91 (2003) 038101.
- [23] B. Lindner, J. García-Ojalvo, A. Neiman, L. Schimansky-Geier, Phys. Rep. 392 (2004) 321.
- [24] A. M. Lacasta, J. M. Sancho, A. H. Romero, K. Lindenberg, Phys. Rev. Lett. 94 (2005) 160601.
- [25] K. Lindenberg, A. M. Lacasta, J. M. Sancho, A. H. Romero, New J. Phys. 7 (2005) 29.
- [26] R. Gautes, S. Miret-Artés, Phys. Rev. E 67 (2003) 046212.
- [27] S. Sengupta, R. Gautes, S. Miret-Artés, P. Hänggi, Physica A 338 (2004) 406.
- [28] C. Reichhardt, C. J. Olson Reichhardt, M. B. Hastings, Phys. Rev. E 69 (2004) 056115.
- [29] S. Savel'ev, V. Misko, F. Marchesoni, F. Nori, Phys. Rev. B 71 (2005) 214303.
- [30] T. Bohr, P. Bak, M. H. Jensen, Phys. Rev. A 30 (1984) 1970.
- [31] M. Borromeo, F. Marchesoni, Chaos 15 (2005) 026110.
- [32] M. Borromeo, G. Costantini, F. Marchesoni, Phys. Rev. Lett. 82 (1999) 2820.
- [33] R. L. Kautz, Rep. Prog. Phys. 59 (1996) 935.
- [34] D. Reguera, P. Reimann, P. Hänggi, M. Rubí, Europhys. Lett. 57 (2002) 644.
- [35] M. Beck, E. Goldobin, M. Neuhaus, M. Siegel, R. Kleiner, D. Koelle, Phys. Rev. Lett. 95 (2005) 090603.
- [36] A. Sterck, R. Kleiner, D. Koelle, Phys. Rev. Lett. 95 (2005) 177006.
- [37] U. E. Vincent, A. N. Njah, O. Akinalde, A. R. T. Solarin, Chaos 14 (2004) 1018.
- [38] U. E. Vincent, A. Kenfack, A. N. Njah, O. Akinalde, Phys. Rev. E 72 (2005) 056213.
- [39] U. E. Vincent, A. N. Njah, O. Akinalde, A. R. T. Solarin, Physica A 360 (2006) 186.

- [40] D. G. Zarlenga, H. A. Larrondo, C. M. Arizmendi, F. Family, *Physica A* 352 (2005) 282.
- [41] M. Kostur, P. Hänggi, P. Talkner, J. L. Mateos, *Phys. Rev. E* 72 (2005) 036210.
- [42] M. H. Jensen, P. Bak, T. Bohr, *Phys. Rev. A* 30 (1984) 1960.
- [43] C. Reichhardt, F. Nori, *Phys. Rev. Lett.* 82 (1999) 414.
- [44] G. V. Osipov, A. S. Pikovsky, J. Kurths, *Phys. Rev. Lett.* 88 (2002) 054102.